

1

適当な公式を用いて、次の式を因数分解せよ。

- (1)  $x^2 + 18x + 81$       (2)  $a^2 - 16ab + 64b^2$       (3)  $9a^2 + 6a + 1$   
 (4)  $25a^2 - 60ab + 36b^2$       (5)  $16 - 8x + x^2$       (6)  $\frac{1}{4}x^2 - x + 1$

2

適当な公式を用いて、次の式を因数分解せよ。

- (1)  $25a^2 - 64$       (2)  $49x^2 - 16y^2$       (3)  $18a^2 - 8b^2$       (4)  $5a^3 - 20ab^2$

3

適当な公式を用いて、次の式を因数分解せよ。

- (1)  $x^2 + 16x + 48$       (2)  $x^2 - 3x - 18$   
 (3)  $x^2 + 2xy - 15y^2$       (4)  $y^2 - 6xy - 16x^2$

4

適当な公式を用いて、次の式を因数分解せよ。

- (1)  $5x^2 + 7x + 2$       (2)  $3a^2 - 10a + 3$       (3)  $2a^2 - 7a - 15$   
 (4)  $3x^2 + 17x + 10$       (5)  $4a^2 + 3a - 27$       (6)  $12x^2 - 25x + 12$   
 (7)  $5a^2 + 7ab - 6b^2$       (8)  $6x^2 - xy - 12y^2$

5

次の式を因数分解せよ。

- (1)  $x^2 - 6x + 9 - y^2$       (2)  $x^2 - y^2 + 4y - 4$   
 (3)  $4x^2 - 4y^2 + 4y - 1$       (4)  $x^2 - 2xy + y^2 - 9z^2$

6

次の式を因数分解せよ。

- (1)  $(x+2)^2 + 5(x+2) + 6$       (2)  $(x-y)^2 - x + y - 12$       (3)  $6(x+y)^2 - 5(x+y) - 4$

(4)  $(x-y)^2 - 5(x-y)z + 4z^2$

(5)  $(a+b)^2 + 8c(a+b) + 16c^2$

(6)  $(x+y+1)(x+y-3) - 12$

(7)  $(x+3y)(x+3y-z) - 6z^2$

7

次の式を因数分解せよ。

- (1)  $x^4 - 13x^2 - 48$       (2)  $x^4 - 1$   
 (3)  $4a^4 - 25a^2b^2 + 36b^4$

8

次の式を因数分解せよ。

- (1)  $18a^2b^2 - 8b^4c^2$       (2)  $4a^2 - \frac{1}{9}(b-c)^2$       (3)  $2x^2 + 28xy - 144y^2$   
 (4)  $x^2 + (a-b)x - ab$       (5)  $15x^2 - 4x - 3$       (6)  $3x^2 + 11xy + 6y^2$

9

適当な公式を用いて、次の式を因数分解せよ。

- (1)  $8x^3 + 1$       (2)  $64x^3 - 27$       (3)  $8x^3 + 27y^3$

10

次の式を因数分解せよ。

- (1)  $abx^2 - (a^2 + b^2)x + ab$       (2)  $abx^2 + (a^2 - b^2)xy - aby^2$

11

次の式を因数分解せよ。

- (1)  $(x^2 - x)^2 - 8(x^2 - x) + 12$       (2)  $(x^2 + 2x)(x^2 + 2x - 2) - 3$   
 (3)  $x^6 + 7x^3 - 8$

12

適当な文字について整理し、次の式を因数分解せよ。

(1)  $xy - x - y + 1$

(2)  $ab + bc - cd - da$

(3)  $25 - 15y + 3xy - x^2$

(4)  $a^2b + a^2 - b - 1$

(5)  $a^2 + b^2 + 2bc + 2ca + 2ab$

(6)  $a^2 + b^2 + bc - ca - 2ab$

13

次の式を因数分解せよ。

(1)  $x^2 + (3y + 1)x + (y + 4)(2y - 3)$

(2)  $x^2 + 3xy + 2y^2 - 6x - 11y + 5$

(3)  $x^2 - 2xy + y^2 - x + y - 2$

(4)  $2x^2 + 5xy + 2y^2 + 4x - y - 6$

(5)  $2x^2 + xy - y^2 + 7x - 5y - 4$

(6)  $2x^2 + 5xy - 3y^2 - x + 11y - 6$

14

次の式を因数分解せよ。

(1)  $a^2(b - c) + b^2(c - a) + c^2(a - b)$

(2)  $(a + b)(b + c)(c + a) + abc$

15

次の式を因数分解せよ。

(1)  $x^4 + 3x^2 + 4$

(2)  $x^4 - 6x^2 + 1$

(3)  $x^4 - 18x^2y^2 + y^4$

(4)  $x^4 + 4y^4$

1

(1) 与式  $= x^2 + 2 \cdot x \cdot 9 + 9^2 = (x+9)^2$

(2) 与式  $= a^2 - 2 \cdot a \cdot 8b + (8b)^2 = (a-8b)^2$

(3) 与式  $= (3a)^2 + 2 \cdot 3a \cdot 1 + 1^2 = (3a+1)^2$

(4) 与式  $= (5a)^2 - 2 \cdot 5a \cdot 6b + (6b)^2 = (5a-6b)^2$

(5) 与式  $= x^2 - 2 \cdot x \cdot 4 + 4^2 = (x-4)^2$

(6) 与式  $= \frac{1}{4}(x^2 - 4x + 4) = \frac{1}{4}(x^2 - 2 \cdot x \cdot 2 + 2^2) = \frac{1}{4}(x-2)^2$

2

(1) 与式  $= (5a)^2 - 8^2 = (5a+8)(5a-8)$

(2) 与式  $= (7x)^2 - (4y)^2 = (7x+4y)(7x-4y)$

(3) 与式  $= 2(9a^2 - 4b^2) = 2[(3a)^2 - (2b)^2] = 2(3a+2b)(3a-2b)$

(4) 与式  $= 5a(a^2 - 4b^2) = 5a[a^2 - (2b)^2] = 5a(a+2b)(a-2b)$

3

(1) 与式  $= x^2 + (4+12)x + 4 \cdot 12 = (x+4)(x+12)$

(2) 与式  $= x^2 + (3-6)x + 3 \cdot (-6) = (x+3)(x-6)$

(3) 与式  $= x^2 + (-3y+5y)x + (-3y) \cdot 5y = (x-3y)(x+5y)$

(4) 与式  $= y^2 + (2x-8x)y + 2x \cdot (-8x) = (y+2x)(y-8x)$

4

(1)  $5x^2 + 7x + 2 = (x+1)(5x+2)$

(2)  $3a^2 - 10a + 3 = (a-3)(3a-1)$

$$\begin{array}{r} 1 \quad \times \quad 1 \rightarrow 5 \\ 5 \quad \times \quad 2 \rightarrow 2 \\ \hline 5 \quad 2 \quad 7 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad -3 \rightarrow -9 \\ 3 \quad \times \quad -1 \rightarrow -1 \\ \hline 3 \quad 3 \quad -10 \end{array}$$

(3)  $2a^2 - 7a - 15 = (a-5)(2a+3)$

(4)  $3x^2 + 17x + 10 = (x+5)(3x+2)$

$$\begin{array}{r} 1 \quad \times \quad -5 \rightarrow -10 \\ 2 \quad \times \quad 3 \rightarrow 3 \\ \hline 2 \quad -15 \quad -7 \end{array}$$

$$\begin{array}{r} 1 \quad \times \quad 5 \rightarrow 15 \\ 3 \quad \times \quad 2 \rightarrow 2 \\ \hline 3 \quad 10 \quad 17 \end{array}$$

(5)  $4a^2 + 3a - 27 = (a+3)(4a-9)$

(6)  $12x^2 - 25x + 12 = (3x-4)(4x-3)$

$$\begin{array}{r} 1 \quad \times \quad 3 \rightarrow 12 \\ 4 \quad \times \quad -9 \rightarrow -9 \\ \hline 4 \quad -27 \quad 3 \end{array}$$

$$\begin{array}{r} 3 \quad \times \quad -4 \rightarrow -12 \\ 4 \quad \times \quad -3 \rightarrow -9 \\ \hline 12 \quad 12 \quad -25 \end{array}$$

(7)  $5a^2 + 7ab - 6b^2 = (a+2b)(5a-3b)$

(8)  $6x^2 - xy - 12y^2 = (2x-3y)(3x+4y)$

$$\begin{array}{r} 1 \quad \times \quad 2b \rightarrow 10b \\ 5 \quad \times \quad -3b \rightarrow -3b \\ \hline 5 \quad -6b^2 \quad 7b \end{array}$$

$$\begin{array}{r} 2 \quad \times \quad -3y \rightarrow -9y \\ 3 \quad \times \quad 4y \rightarrow 8y \\ \hline 6 \quad -12y^2 \quad -y \end{array}$$

5

(1) 与式  $= (x^2 - 6x + 9) - y^2 = (x-3)^2 - y^2 = \{(x-3)+y\}\{(x-3)-y\}$   
 $= (x+y-3)(x-y-3)$

(2) 与式  $= x^2 - (y^2 - 4y + 4) = x^2 - (y-2)^2 = \{x+(y-2)\}\{x-(y-2)\}$   
 $= (x+y-2)(x-y+2)$

(3) 与式  $= 4x^2 - (4y^2 - 4y + 1) = (2x)^2 - (2y-1)^2 = \{2x+(2y-1)\}\{2x-(2y-1)\}$   
 $= (2x+2y-1)(2x-2y+1)$

(4) 与式  $= (x^2 - 2xy + y^2) - 9z^2 = (x-y)^2 - (3z)^2 = \{(x-y)+3z\}\{(x-y)-3z\}$   
 $= (x-y+3z)(x-y-3z)$

6

(1) 与式  $= \{(x+2)+2\}\{(x+2)+3\} = (x+4)(x+5)$

(2) 与式  $= (x-y)^2 - (x-y) - 12 = \{(x-y)+3\}\{(x-y)-4\} = (x-y+3)(x-y-4)$

(3) 与式  $= \{2(x+y)+1\}\{3(x+y)-4\}$   
 $= (2x+2y+1)(3x+3y-4)$

$$\begin{array}{r} 2 \quad \times \quad 1 \rightarrow 3 \\ 3 \quad \times \quad -4 \rightarrow -8 \\ \hline 6 \quad -4 \quad -5 \end{array}$$

(4) 与式  $= \{(x-y)-z\}\{(x-y)-4z\}$   
 $= (x-y-z)(x-y-4z)$

(5) 与式  $= \{(a+b)+4c\}^2 = (a+b+4c)^2$

(6) 与式  $= (x+y)^2 - 2(x+y) - 15 = \{(x+y)+3\}\{(x+y)-5\} = (x+y+3)(x+y-5)$

(7) 与式  $= (x+3y)^2 - z(x+3y) - 6z^2 = \{(x+3y)+2z\}\{(x+3y)-3z\}$   
 $= (x+3y+2z)(x+3y-3z)$

7

(1) 与式  $= (x^2)^2 - 13x^2 - 48 = (x^2 - 16)(x^2 + 3) = (x + 4)(x - 4)(x^2 + 3)$

(2) 与式  $= (x^2)^2 - 1 = (x^2 - 1)(x^2 + 1) = (x + 1)(x - 1)(x^2 + 1)$

(3) 与式  $= 4(a^2)^2 - 25a^2b^2 + 36(b^2)^2 = (a^2 - 4b^2)(4a^2 - 9b^2)$   
 $= (a + 2b)(a - 2b)(2a + 3b)(2a - 3b)$

1	×	-4	→	-16
4	×	-9	→	-9
4		36		-25

8

(1) 与式  $= 2b^2(9a^2 - 4b^2c^2) = 2b^2\{(3a)^2 - (2bc)^2\} = 2b^2(3a + 2bc)(3a - 2bc)$

(2) 与式  $= \frac{1}{9}\{36a^2 - (b - c)^2\} = \frac{1}{9}\{(6a)^2 - (b - c)^2\} = \frac{1}{9}\{6a + (b - c)\}\{6a - (b - c)\}$   
 $= \frac{1}{9}(6a + b - c)(6a - b + c)$

(3) 与式  $= 2(x^2 + 14xy - 72y^2) = 2\{x^2 + (18y - 4y)x + 18y(-4y)\} = 2(x + 18y)(x - 4y)$

(4) 与式  $= (x + a)(x - b)$

(5) 与式  $= (3x + 1)(5x - 3)$

(6) 与式  $= (x + 3y)(3x + 2y)$

3	×	1	→	5
5	×	-3	→	-9
15		-3		-4

1	×	3y	→	9y
3	×	2y	→	2y
3		6y <sup>2</sup>		11y

9

(1) 与式  $= (2x)^3 + 1^3 = (2x + 1)\{(2x)^2 - 2x \cdot 1 + 1^2\} = (2x + 1)(4x^2 - 2x + 1)$

(2) 与式  $= (4x)^3 - 3^3 = (4x - 3)\{(4x)^2 + 4x \cdot 3 + 3^2\} = (4x - 3)(16x^2 + 12x + 9)$

(3) 与式  $= (2x)^3 + (3y)^3 = (2x + 3y)\{(2x)^2 - 2x \cdot 3y + (3y)^2\} = (2x + 3y)(4x^2 - 6xy + 9y^2)$

10

(1) 与式  $= (ax - b)(bx - a)$

a	×	-b	→	-b <sup>2</sup>
b	×	-a	→	-a <sup>2</sup>
ab		ab		-(a <sup>2</sup> + b <sup>2</sup> )

(2) 与式  $= (ax - by)(bx + ay)$

a	×	-by	→	-b <sup>2</sup> y
b	×	ay	→	a <sup>2</sup> y
ab		-aby <sup>2</sup>		(a <sup>2</sup> - b <sup>2</sup> )y

11

(1) 与式  $= \{(x^2 - x) - 2\}\{(x^2 - x) - 6\} = (x^2 - x - 2)(x^2 - x - 6) = (x + 1)(x - 2)(x + 2)(x - 3)$

(2) 与式  $= (x^2 + 2x)^2 - 2(x^2 + 2x) - 3 = \{(x^2 + 2x) + 1\}\{(x^2 + 2x) - 3\}$   
 $= (x^2 + 2x + 1)(x^2 + 2x - 3) = (x + 1)^2(x - 1)(x + 3)$

(3) 与式  $= (x^3)^2 + 7x^3 - 8 = (x^3 - 1)(x^3 + 8)$   
 $= (x - 1)(x^2 + x + 1)(x + 2)(x^2 - 2x + 4)$   
 $= (x - 1)(x + 2)(x^2 + x + 1)(x^2 - 2x + 4)$

12

(1) 与式  $= x(y - 1) - y + 1 = x(y - 1) - (y - 1) = (x - 1)(y - 1)$

(2) 与式  $= a(b - d) + bc - cd = a(b - d) + c(b - d) = (a + c)(b - d)$

(3) 与式  $= 3(x - 5)y - (x^2 - 25) = 3(x - 5)y - (x + 5)(x - 5) = (x - 5)(-x + 3y - 5)$

(4) 与式  $= (a^2 - 1)b + (a^2 - 1) = (a^2 - 1)(b + 1) = (a + 1)(a - 1)(b + 1)$

(5) 与式  $= 2(a + b)c + (a^2 + 2ab + b^2) = 2(a + b)c + (a + b)^2 = (a + b)(a + b + 2c)$

(6) 与式  $= (b - a)c + (a^2 - 2ab + b^2) = -(a - b)c + (a - b)^2 = (a - b)(a - b - c)$

13

(1) 与式  $= \{x + (y + 4)\}\{x + (2y - 3)\}$   
 $= (x + y + 4)(x + 2y - 3)$

$$\begin{array}{r} 1 \quad \times \quad y+4 \longrightarrow y+4 \\ 1 \quad \times \quad 2y-3 \longrightarrow 2y-3 \\ \hline 3y+1 \end{array}$$

(2) 与式  $= x^2 + (3y - 6)x + (2y^2 - 11y + 5)$   
 $= x^2 + (3y - 6)x + (2y - 1)(y - 5)$   
 $= \{x + (2y - 1)\}\{x + (y - 5)\}$   
 $= (x + 2y - 1)(x + y - 5)$

$$\begin{array}{r} 1 \quad \times \quad 2y-1 \longrightarrow 2y-1 \\ 1 \quad \times \quad y-5 \longrightarrow y-5 \\ \hline 3y-6 \end{array}$$

(3) 与式  $= x^2 - (2y + 1)x + y^2 + y - 2$   
 $= x^2 - (2y + 1)x + (y - 1)(y + 2)$   
 $= \{x - (y - 1)\}\{x - (y + 2)\}$   
 $= (x - y + 1)(x - y - 2)$

$$\begin{array}{r} 1 \quad \times \quad -(y-1) \longrightarrow -y+1 \\ 1 \quad \times \quad -(y+2) \longrightarrow -y-2 \\ \hline -2y-1 \end{array}$$

別解 与式  $= (x - y)^2 - (x - y) - 2 = \{(x - y) + 1\}\{(x - y) - 2\} = (x - y + 1)(x - y - 2)$

(4) 与式  $= 2x^2 + (5y + 4)x + 2y^2 - y - 6$   
 $= 2x^2 + (5y + 4)x + (2y + 3)(y - 2)$   
 $= \{x + (2y + 3)\}\{2x + (y - 2)\}$   
 $= (x + 2y + 3)(2x + y - 2)$

$$\begin{array}{r} 1 \quad \times \quad 2y+3 \longrightarrow 4y+6 \\ 2 \quad \times \quad y-2 \longrightarrow y-2 \\ \hline 5y+4 \end{array}$$

(5) 与式  $= 2x^2 + (y + 7)x - (y^2 + 5y + 4)$   
 $= 2x^2 + (y + 7)x - (y + 1)(y + 4)$   
 $= \{x + (y + 4)\}\{2x - (y + 1)\}$   
 $= (x + y + 4)(2x - y - 1)$

$$\begin{array}{r} 1 \quad \times \quad y+4 \longrightarrow 2y+8 \\ 2 \quad \times \quad -(y+1) \longrightarrow -y-1 \\ \hline y+7 \end{array}$$

(6) 与式  $= 2x^2 + (5y - 1)x - (3y^2 - 11y + 6)$   
 $= 2x^2 + (5y - 1)x - (3y - 2)(y - 3)$   
 $= \{x + (3y - 2)\}\{2x - (y - 3)\}$   
 $= (x + 3y - 2)(2x - y + 3)$

$$\begin{array}{r} 1 \quad \times \quad 3y-2 \longrightarrow 6y-4 \\ 2 \quad \times \quad -(y-3) \longrightarrow -y+3 \\ \hline 5y-1 \end{array}$$

14

(1) 与式  $= (b - c)a^2 - (b^2 - c^2)a + b^2c - bc^2 = (b - c)a^2 - (b - c)(b + c)a + bc(b - c)$   
 $= (b - c)\{a^2 - (b + c)a + bc\} = (b - c)(a - b)(a - c) = -(a - b)(b - c)(c - a)$

(2) 与式  $= \{(a + b)(a + c)\}(b + c) + abc = \{a^2 + (b + c)a + bc\}(b + c) + abc$

$$\begin{aligned} &= (b + c)a^2 + (b + c)^2a + bc(b + c) + abc \\ &= (b + c)a^2 + \{(b + c)^2 + bc\}a + bc(b + c) \\ &= \{a + (b + c)\}\{(b + c)a + bc\} \\ &= (a + b + c)(ab + bc + ca) \end{aligned}$$

$$\begin{array}{r} 1 \quad \times \quad b+c \longrightarrow (b+c)^2 \\ b+c \quad \times \quad bc \longrightarrow bc \\ \hline b+c \quad bc(b+c) \quad (b+c)^2+bc \end{array}$$

15

(1) 与式  $= (x^4 + 4x^2 + 4) - x^2 = (x^2 + 2)^2 - x^2 = \{(x^2 + 2) + x\}\{(x^2 + 2) - x\}$   
 $= (x^2 + x + 2)(x^2 - x + 2)$

(2) 与式  $= (x^4 - 2x^2 + 1) - 4x^2 = (x^2 - 1)^2 - (2x)^2 = \{(x^2 - 1) + 2x\}\{(x^2 - 1) - 2x\}$   
 $= (x^2 + 2x - 1)(x^2 - 2x - 1)$

(3) 与式  $= (x^4 - 2x^2y^2 + y^4) - 16x^2y^2 = (x^2 - y^2)^2 - (4xy)^2$   
 $= \{(x^2 - y^2) + 4xy\}\{(x^2 - y^2) - 4xy\} = (x^2 + 4xy - y^2)(x^2 - 4xy - y^2)$

(4) 与式  $= (x^4 + 4x^2y^2 + 4y^4) - 4x^2y^2 = (x^2 + 2y^2)^2 - (2xy)^2$   
 $= \{(x^2 + 2y^2) + 2xy\}\{(x^2 + 2y^2) - 2xy\} = (x^2 + 2xy + 2y^2)(x^2 - 2xy + 2y^2)$