

1

次の循環小数を分数で表せ。

- (1)  $0.\dot{4}$                       (2)  $0.\dot{7}\dot{9}$                       (3)  $0.\dot{4}5\dot{6}$                       (4)  $-3.9\dot{7}\dot{2}$

2

$x = -4, -1, 2, 5$  のそれぞれについて、次の式の値を求めよ。

- (1)  $|x|$                                       (2)  $|x+1|$                                       (3)  $|1-2x|+|x-1|$

3

次の式を計算せよ。

- (1)  $3\sqrt{3} + \sqrt{75} - \sqrt{48}$                       (2)  $2\sqrt{7} - \sqrt{63} + \sqrt{28}$                       (3)  $\sqrt{3}(2\sqrt{3} - \sqrt{6})$   
 (4)  $\sqrt{5}(3\sqrt{10} - 2\sqrt{5})$                       (5)  $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3})$                       (6)  $(\sqrt{20} + \sqrt{3})(\sqrt{5} - \sqrt{27})$   
 (7)  $(\sqrt{3} + \sqrt{5})^2$                                       (8)  $(2\sqrt{3} - 3\sqrt{2})^2$

4

次の式の分母を有理化せよ。

- (1)  $\frac{2}{\sqrt{5}}$                       (2)  $\frac{4}{3\sqrt{8}}$                       (3)  $\frac{1}{\sqrt{2}+1}$                       (4)  $\frac{1}{\sqrt{5}-\sqrt{3}}$   
 (5)  $\frac{2+\sqrt{3}}{2-\sqrt{3}}$                       (6)  $\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$                       (7)  $\frac{\sqrt{3}-1}{2\sqrt{3}-5}$                       (8)  $\frac{2\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

5

次の式を計算せよ。

- (1)  $(1+\sqrt{2}-\sqrt{3})^2$                                       (2)  $(3-\sqrt{2}-\sqrt{11})(3-\sqrt{2}+\sqrt{11})$

6

次の式を計算せよ。

- (1)  $\frac{3\sqrt{5}-5\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{3\sqrt{5}+4\sqrt{3}}{3\sqrt{5}-4\sqrt{3}}$                       (2)  $\frac{\sqrt{2}-1}{\sqrt{2}+1} + \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{2-\sqrt{3}}$

7

$x = \frac{\sqrt{5}+2}{\sqrt{5}-2}$ ,  $y = \frac{\sqrt{5}-2}{\sqrt{5}+2}$  のとき、次の値を求めよ。

- (1)  $x+y$                       (2)  $xy$                       (3)  $x^2y+xy^2$                       (4)  $x^2+y^2$                       (5)  $x^3+y^3$

8

$x = \sqrt{2}-1$  のとき、次の値を求めよ。

- (1)  $x + \frac{1}{x}$                       (2)  $x^2 + \frac{1}{x^2}$                       (3)  $x^3 + \frac{1}{x^3}$                       (4)  $x^5 + \frac{1}{x^5}$

9

$x = 1 - \sqrt{5}$  のとき、次の値を求めよ。

- (1)  $x^2 - 2x - 4$                                       (2)  $x^3 - 2x^2$

10

$\frac{\sqrt{2}}{\sqrt{2}-1}$  の整数部分を  $a$ , 小数部分を  $b$  とする。次の値を求めよ。

- (1)  $a$                                       (2)  $b$                                       (3)  $a+b+b^2$

11

次の式を計算せよ。

- (1)  $\frac{1}{1+\sqrt{2}-\sqrt{3}}$                                       (2)  $\frac{\sqrt{5}+\sqrt{3}+\sqrt{2}}{\sqrt{5}+\sqrt{3}-\sqrt{2}}$   
 (3)  $\frac{\sqrt{2}+\sqrt{5}+\sqrt{7}}{\sqrt{2}+\sqrt{5}-\sqrt{7}} + \frac{\sqrt{2}-\sqrt{5}+\sqrt{7}}{\sqrt{2}-\sqrt{5}-\sqrt{7}}$

12

次の各場合について、 $\sqrt{x^2 - 10x + 25}$  を  $x$  の整式で表せ。

(1)  $x - 5 \geq 0$

(2)  $x - 5 < 0$

13

次の式を簡単にせよ。

(1)  $\sqrt{4 + 2\sqrt{3}}$

(2)  $\sqrt{19 - 2\sqrt{48}}$

(3)  $\sqrt{9 - 2\sqrt{20}}$

14

次の式を簡単にせよ。

(1)  $\sqrt{5 + \sqrt{24}}$

(2)  $\sqrt{11 + 4\sqrt{6}}$

(3)  $\sqrt{12 - 8\sqrt{2}}$

15

次の式を簡単にせよ。

(1)  $\sqrt{2 + \sqrt{3}}$

(2)  $\sqrt{5 - \sqrt{21}}$

(3)  $\sqrt{10 + 5\sqrt{3}}$

1

- (1)  $x=0.\dot{4}$  とおくと  $10x=4.\dot{4}$  よって  $10x-x=4$   
 ゆえに  $x=\frac{4}{9}$  すなわち  $0.\dot{4}=\frac{4}{9}$
- (2)  $x=0.\dot{7}\dot{9}$  とおくと  $100x=79.\dot{7}\dot{9}$  よって  $100x-x=79$   
 ゆえに  $x=\frac{79}{99}$  すなわち  $0.\dot{7}\dot{9}=\frac{79}{99}$
- (3)  $x=0.\dot{4}5\dot{6}$  とおくと  $1000x=456.\dot{4}5\dot{6}$  よって  $1000x-x=456$   
 ゆえに  $x=\frac{456}{999}=\frac{152}{333}$  すなわち  $0.\dot{4}5\dot{6}=\frac{152}{333}$
- (4)  $x=-3.9\dot{7}\dot{2}$  とおくと  $10x=-39.\dot{7}\dot{2}$ ,  $1000x=-3972.\dot{7}\dot{2}$   
 よって  $1000x-10x=-3933$   
 ゆえに  $x=\frac{-3933}{990}=-\frac{437}{110}$  すなわち  $-3.9\dot{7}\dot{2}=-\frac{437}{110}$

2

$x$  に  $-4, -1, 2, 5$  を順に代入する。

- (1)  $|-4|=4, |-1|=1, |2|=2, |5|=5$
- (2)  $|-4+1|=|-3|=3, |-1+1|=|0|=0, |2+1|=|3|=3, |5+1|=|6|=6$
- (3)  $|1-2\cdot(-4)|+|-4-1|=|9|+|-5|=9+5=14$   
 $|1-2\cdot(-1)|+|-1-1|=|3|+|-2|=3+2=5$   
 $|1-2\cdot2|+|2-1|=|-3|+|1|=3+1=4$   
 $|1-2\cdot5|+|5-1|=|-9|+|4|=9+4=13$

3

- (1) 与式  $=3\sqrt{3}+5\sqrt{3}-4\sqrt{3}=4\sqrt{3}$
- (2) 与式  $=2\sqrt{7}-3\sqrt{7}+2\sqrt{7}=\sqrt{7}$
- (3) 与式  $=2\cdot 3-\sqrt{3^2\cdot 2}=6-3\sqrt{2}$
- (4) 与式  $=3\cdot 5\sqrt{2}-2\cdot 5=15\sqrt{2}-10$
- (5) 与式  $=(\sqrt{5})^2-(\sqrt{3})^2=5-3=2$
- (6) 与式  $=(2\sqrt{5}+\sqrt{3})(\sqrt{5}-3\sqrt{3})=10-6\sqrt{15}+\sqrt{15}-3\cdot 3=1-5\sqrt{15}$
- (7) 与式  $=3+2\sqrt{15}+5=8+2\sqrt{15}$

(8) 与式  $=12-2\cdot 6\sqrt{6}+18=30-12\sqrt{6}$

4

- (1) 与式  $=\frac{2\sqrt{5}}{5}$  (2) 与式  $=\frac{4}{3\cdot 2\sqrt{2}}=\frac{2}{3\sqrt{2}}=\frac{2\sqrt{2}}{3\cdot 2}=\frac{\sqrt{2}}{3}$
- (3) 与式  $=\frac{\sqrt{2}-1}{(\sqrt{2}+1)(\sqrt{2}-1)}=\frac{\sqrt{2}-1}{(\sqrt{2})^2-1^2}=\sqrt{2}-1$
- (4) 与式  $=\frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}=\frac{\sqrt{5}+\sqrt{3}}{(\sqrt{5})^2-(\sqrt{3})^2}=\frac{\sqrt{5}+\sqrt{3}}{2}$
- (5) 与式  $=\frac{(2+\sqrt{3})^2}{(2-\sqrt{3})(2+\sqrt{3})}=\frac{4+4\sqrt{3}+3}{2^2-(\sqrt{3})^2}=7+4\sqrt{3}$
- (6) 与式  $=\frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}=\frac{5-2\sqrt{10}+2}{(\sqrt{5})^2-(\sqrt{2})^2}=\frac{7-2\sqrt{10}}{3}$
- (7) 与式  $=\frac{(\sqrt{3}-1)(2\sqrt{3}+5)}{(2\sqrt{3}-5)(2\sqrt{3}+5)}=\frac{6+5\sqrt{3}-2\sqrt{3}-5}{(2\sqrt{3})^2-5^2}=-\frac{1+3\sqrt{3}}{13}$
- (8) 与式  $=\frac{(2\sqrt{2}-\sqrt{3})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}=\frac{2\sqrt{6}-4-3+\sqrt{6}}{(\sqrt{3})^2-(\sqrt{2})^2}=-7+3\sqrt{6}$

5

- (1) 与式  $=1^2+(\sqrt{2})^2+(-\sqrt{3})^2+2\cdot 1\cdot \sqrt{2}+2\cdot \sqrt{2}\cdot (-\sqrt{3})+2\cdot (-\sqrt{3})\cdot 1$   
 $=1+2+3+2\sqrt{2}-2\sqrt{6}-2\sqrt{3}=6+2\sqrt{2}-2\sqrt{3}-2\sqrt{6}$
- (2) 与式  $=\{(3-\sqrt{2})-\sqrt{11}\}\{(3-\sqrt{2})+\sqrt{11}\}=(3-\sqrt{2})^2-(\sqrt{11})^2=9-6\sqrt{2}+2-11$   
 $=-6\sqrt{2}$

6

- (1) 与式  $=\frac{(3\sqrt{5}-5\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}+\frac{(3\sqrt{5}+4\sqrt{3})^2}{(3\sqrt{5}-4\sqrt{3})(3\sqrt{5}+4\sqrt{3})}$   
 $=\frac{15-3\sqrt{15}-5\sqrt{15}+15}{5-3}+\frac{45+24\sqrt{15}+48}{45-48}=\frac{30-8\sqrt{15}}{2}-\frac{93+24\sqrt{15}}{3}$   
 $=(15-4\sqrt{15})-(31+8\sqrt{15})=-16-12\sqrt{15}$
- (2) 与式  $=\frac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}+\frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}+\frac{(\sqrt{3}+\sqrt{2})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$   
 $=(3-2\sqrt{2})+(5-2\sqrt{6})+(2\sqrt{3}+3+2\sqrt{2}+\sqrt{6})=11+2\sqrt{3}-\sqrt{6}$

7

$$(1) x + y = \frac{\sqrt{5} + 2}{\sqrt{5} - 2} + \frac{\sqrt{5} - 2}{\sqrt{5} + 2} = \frac{(\sqrt{5} + 2)^2 + (\sqrt{5} - 2)^2}{(\sqrt{5} - 2)(\sqrt{5} + 2)} = (9 + 4\sqrt{5}) + (9 - 4\sqrt{5}) = 18$$

$$(2) xy = \frac{\sqrt{5} + 2}{\sqrt{5} - 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} + 2} = 1$$

$$(3) x^2y + xy^2 = xy(x + y) = 1 \cdot 18 = 18$$

$$(4) x^2 + y^2 = (x + y)^2 - 2xy = 18^2 - 2 \cdot 1 = 322$$

$$(5) x^3 + y^3 = (x + y)^3 - 3xy(x + y) = 18^3 - 3 \cdot 1 \cdot 18 = 5778$$

**別解**  $x^3 + y^3 = (x + y)(x^2 - xy + y^2) = 18(322 - 1) = 5778$

**参考**  $(x + y)^2 = x^2 + 2xy + y^2$  から  $x^2 + y^2 = (x + y)^2 - 2xy$   
 $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$  から  $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

8

$$(1) \frac{1}{x} = \frac{1}{\sqrt{2} - 1} = \frac{\sqrt{2} + 1}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \sqrt{2} + 1$$

よって  $x + \frac{1}{x} = (\sqrt{2} - 1) + (\sqrt{2} + 1) = 2\sqrt{2}$

$$(2) x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = (2\sqrt{2})^2 - 2 = 6$$

$$(3) x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) = (2\sqrt{2})^3 - 3 \cdot 2\sqrt{2} = 10\sqrt{2}$$

**別解**  $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 - x \cdot \frac{1}{x} + \frac{1}{x^2}\right) = 2\sqrt{2}(6 - 1) = 10\sqrt{2}$

$$(4) x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 10\sqrt{2} \cdot 6 - 2\sqrt{2} = 58\sqrt{2}$$

**参考**  $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}$  から  $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + 3 \cdot x^2 \cdot \frac{1}{x} + 3 \cdot x \cdot \frac{1}{x^2} + \frac{1}{x^3}$$
 から  $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$

$$\left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) = x^5 + x^3 \cdot \frac{1}{x^2} + x^2 \cdot \frac{1}{x^3} + \frac{1}{x^5} = x^5 + \frac{1}{x^5} + x + \frac{1}{x}$$
 から

$$x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right)$$

9

$$(1) \text{与式} = (1 - \sqrt{5})^2 - 2(1 - \sqrt{5}) - 4 = (1 - 2\sqrt{5} + 5) - 2 + 2\sqrt{5} - 4 = 0$$

**別解**  $x = 1 - \sqrt{5}$  から  $x - 1 = -\sqrt{5}$

両辺を 2 乗すると  $x^2 - 2x + 1 = 5$  よって  $x^2 - 2x - 4 = 0$

$$(2) (1) \text{から } x^2 - 2x = 4$$

よって  $x^3 - 2x^2 = x(x^2 - 2x) = (1 - \sqrt{5}) \cdot 4 = 4 - 4\sqrt{5}$

10

$$\frac{\sqrt{2}}{\sqrt{2} - 1} = \frac{\sqrt{2}(\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)} = \frac{2 + \sqrt{2}}{(\sqrt{2})^2 - 1^2} = 2 + \sqrt{2}$$

$$(1) 1 < \sqrt{2} < 2 \text{ であるから } 3 < 2 + \sqrt{2} < 4$$

よって  $a = 3$

$$(2) b = (2 + \sqrt{2}) - a = (2 + \sqrt{2}) - 3 = \sqrt{2} - 1$$

$$(3) a + b + b^2 = a + b(1 + b) = 3 + (\sqrt{2} - 1) \cdot \sqrt{2} = 3 + 2 - \sqrt{2} = 5 - \sqrt{2}$$

**別解** もとの数は  $a + b$  であるから

$$a + b + b^2 = (a + b) + b^2 = (2 + \sqrt{2}) + (\sqrt{2} - 1)^2 = 5 - \sqrt{2}$$

11

$$(1) \text{与式} = \frac{1 + \sqrt{2} + \sqrt{3}}{(1 + \sqrt{2} - \sqrt{3})(1 + \sqrt{2} + \sqrt{3})} = \frac{1 + \sqrt{2} + \sqrt{3}}{(1 + \sqrt{2})^2 - (\sqrt{3})^2} = \frac{1 + \sqrt{2} + \sqrt{3}}{2\sqrt{2}}$$

$$= \frac{2 + \sqrt{2} + \sqrt{6}}{4}$$

$$(2) \text{与式} = \frac{(\sqrt{5} + \sqrt{3} + \sqrt{2})(\sqrt{5} - \sqrt{3} + \sqrt{2})}{(\sqrt{5} + \sqrt{3} - \sqrt{2})(\sqrt{5} - \sqrt{3} + \sqrt{2})} = \frac{(\sqrt{5} + \sqrt{2})^2 - (\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3} - \sqrt{2})^2} = \frac{4 + 2\sqrt{10}}{2\sqrt{6}}$$

$$= \frac{\sqrt{6} + \sqrt{15}}{3}$$

$$(3) \text{与式} = \frac{(\sqrt{2} + \sqrt{5} + \sqrt{7})^2}{(\sqrt{2} + \sqrt{5} - \sqrt{7})(\sqrt{2} + \sqrt{5} + \sqrt{7})} + \frac{(\sqrt{2} - \sqrt{5} + \sqrt{7})^2}{(\sqrt{2} - \sqrt{5} - \sqrt{7})(\sqrt{2} - \sqrt{5} + \sqrt{7})}$$

$$= \frac{14 + 2\sqrt{10} + 2\sqrt{35} + 2\sqrt{14}}{(\sqrt{2} + \sqrt{5})^2 - (\sqrt{7})^2} + \frac{14 - 2\sqrt{10} - 2\sqrt{35} + 2\sqrt{14}}{(\sqrt{2} - \sqrt{5})^2 - (\sqrt{7})^2} = \frac{4\sqrt{10} + 4\sqrt{35}}{2\sqrt{10}}$$

$$= 2 + \sqrt{14}$$

12

$$\sqrt{x^2 - 10x + 25} = \sqrt{(x-5)^2} = |x-5|$$

- (1)  $x-5 \geq 0$  であるから 与式  $= x-5$   
 (2)  $x-5 < 0$  であるから 与式  $= -(x-5) = -x+5$

13

- (1) 与式  $= \sqrt{(3+1)+2\sqrt{3 \cdot 1}} = \sqrt{3} + \sqrt{1} = \sqrt{3} + 1$   
 (2) 与式  $= \sqrt{(16+3)-2\sqrt{16 \cdot 3}} = \sqrt{16} - \sqrt{3} = 4 - \sqrt{3}$   
 (3) 与式  $= \sqrt{(5+4)-2\sqrt{5 \cdot 4}} = \sqrt{5} - \sqrt{4} = \sqrt{5} - 2$

14

- (1) 与式  $= \sqrt{5+2\sqrt{6}} = \sqrt{(3+2)+2\sqrt{3 \cdot 2}} = \sqrt{3} + \sqrt{2}$   
 (2) 与式  $= \sqrt{11+2\sqrt{24}} = \sqrt{(8+3)+2\sqrt{8 \cdot 3}} = \sqrt{8} + \sqrt{3} = 2\sqrt{2} + \sqrt{3}$   
 (3) 与式  $= \sqrt{12-2\sqrt{32}} = \sqrt{(8+4)-2\sqrt{8 \cdot 4}} = \sqrt{8} - \sqrt{4} = 2\sqrt{2} - 2$

15

- (1) 与式  $= \sqrt{\frac{4+2\sqrt{3}}{2}} = \frac{\sqrt{(3+1)+2\sqrt{3 \cdot 1}}}{\sqrt{2}} = \frac{\sqrt{3} + \sqrt{1}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{2}$   
 (2) 与式  $= \sqrt{\frac{10-2\sqrt{21}}{2}} = \frac{\sqrt{(7+3)-2\sqrt{7 \cdot 3}}}{\sqrt{2}} = \frac{\sqrt{7} - \sqrt{3}}{\sqrt{2}} = \frac{\sqrt{14} - \sqrt{6}}{2}$   
 (3) 与式  $= \sqrt{10 + \sqrt{75}} = \sqrt{\frac{20 + 2\sqrt{75}}{2}} = \frac{\sqrt{(15+5)+2\sqrt{15 \cdot 5}}}{\sqrt{2}} = \frac{\sqrt{15} + \sqrt{5}}{\sqrt{2}}$   

$$= \frac{\sqrt{30} + \sqrt{10}}{2}$$